3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Physics

Course : PHYS1021 (MINOR)

(Mathematical Physics-I)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols have their usual meaning.

Group-A

1. Answer *any five* questions from the following: 2×5=10

(a) What do you mean by limit of a function? Find the value of $\lim_{x\to 0}\frac{\tan x}{x}$.

(b) For a function $f(x) = \frac{(x+1)(x^2-25)}{(x-1)(5x^2+30+45)}$, list all values of x at which $f(x)$ is not defined.

(c) What is Wronskian of a differential equation? Show that if two solutions are linearly independent Wronskian vanishes.

(d) Find the projection of the vector $\vec{A} = 3\hat{\imath} + 3\hat{\jmath} - \hat{k}$ on the vector $\vec{B} = 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$.

(e) If $\phi(x,y,z) = 3x^2y - y^2z + xz^2$, find $\vec{\nabla}\phi$ at the point (3, 1, –1).

(f) Determine the constant α so that the vector $\vec{A} = (2x+3y)\hat{\imath} + (y-2z)\hat{\jmath} + (3x+2\alpha z)\hat{k}$ is solenoidal.

(g) Evaluate $\iint_S \vec{F}.\hat{n}\,dS$, where $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and S is the surface bounded by a unit cube.

(h) Solve the differential equation $\frac{dy}{dx} + 2y = 3e^x$.

Group-B

Answer *any two* questions from the following: 5×2=10

2. State the Stokes' theorem in words. Verify Stokes' theorem for $\vec{A} = (2x-y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. 2+3

3. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time.

(a) Determine its acceleration at any time.

(b) Find the magnitudes of the velocity and acceleration at $t = 0$. 2+3

4. Obtain the scale factors for the cylindrical constants. Show that the cylindrical coordinates are orthogonal coordinate system. 3+2

5. State the existence and uniqueness theorem for initial value problems of ordinary differential equations. A tank initially contains 50 gallon of water. A salt solution containing 2 kg of salt per gallon water is poured into the tank at a rate of 3 gallon/minute. The mixture is stirred and is drained out the tank at the same rate. Find the initial-value problem that describes the amount Q of salt in the tank at any time. 2+3

Group-C

Answer *any two* questions from the following: 10×2=20

6. Define the irrotational vector. Find constants a, b, c so that a vector $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - 2z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$, where n is a constant. 2+2+6

7. What do you mean by linear differential equations? What is complementary function and particular integral? Solve the differential equation $\frac{d^2y}{dx^2} - 4y = x \sin hx$. 2+2+6

8. What do you mean by vector field? A vector field $\vec{F}$ is given by $\vec{F} = xy^2\hat{\imath} + 2\hat{\jmath} + x\hat{k}$ and L is a path parameterized by $x = ct$, $y = c/t$, $z = d$ for the range $1 \le t \le 2$. Evaluate the three integrals

(a) $\int_L \vec{F} dt$

(b) $\int_L \vec{F} dy$

(c) $\int_L \vec{F}.d\vec{r}$

1+3+3+3

9. Plot the function $y = x^2 + x - 1$ for $|x| < 5$. What is an exart differential equation? Show that the differential function $u(x,t) = f(x + ct) + \phi(x - ct)$ satisfies the differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$. 3+2+5